Generalizing Game Theory

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Abstract

Game theoretic concepts apply whenever the actions of several agents are interdependent. These agents may be individuals, groups, firms, or any combination of these. The concepts of game theory provide a language to formulate, structure and analyze situations of interdependence between adaptive agents, out of which can emerge dynamics of conflict and cooperation.

As a method of applied mathematics, game theory has been used to study a wide variety of human and animal behaviors. It was firstly developed in economics to understand a variety of economic behaviors, including behaviors of firms, markets, and consumers, but has since been used to model a myriad of phenomena, such as the interaction of nation states within international politics, crime, ecosystem dynamics, the routing of internet traffic and much more.

Classical game theory describes the behavior of rational players. It attempts to mathematically capture behavior in strategic situations, in which an individual's success in making choices depends on the choices of others.[1] The limitations of this classical game theory[2] are long since know, but they are becoming more apparent as behavioral economics exposes them to real-world experiments and as we attempt to construct a more complex understanding of human motives and human interaction. The game theory of the past was focused on theory, mathematical elegance, closed form solutions, rigor and precise predictions, but in the process, it often ended up divorcing itself from the empirics of real-world socio-economic interaction.

Today developing models for the interaction between people is of critical importance within a wide variety of domains, both in the social sciences but also in management and design. Today there is a great demand for a game theory that puts empirics first; one that is grounded in real-world interaction and can accurately reflect the complexity of the many social interactions that we encounter on a daily basis and the motives of agents engaged in them. Achieving such a game theory framework will require a generalization of the existing game theory framework beyond its foundation.[3]

As we move into the world of studying more complex systems there is an increasing demand for a more general conception of game theory. One that is capable of studying all forms of interaction between adaptive agents without the restrictive assumptions that are an inherent part of Classical Game Theory.

In this paper, we look at the ongoing evolution of game theory, from its origins in classical game theory to more recent developments. We trace the past, present and potential future of game theory in search of an expanded formulation of game theory that would be relevant to the modeling of complex socioeconomic systems. Our primary focus here is on identifying the limitations within the existing game theory framework and how it can be expanded to accurately deal with more complex scenarios of socioeconomic interaction. At each step we try to identify and make explicit the assumptions inherent in standard game theory models and how these both enable and constrain the modeling process; how such assumptions may be revised in the light of new developments within complexity theory to better suit the modeling of interactions within complex socioeconomic systems.

Evolution of Game Theory

Like all modeling frameworks, game theory started simple and is evolving to become more complex. "Starting simple" meant making many simplifying assumptions about human decision making and scenarios of interaction. These simplifying assumptions were imposed in order for the phenomena under consideration to fit into the mathematical modeling paradigm of the time; as we will illustrate in this paper.

This paper assumes that the reader is largely familiar with the workings of standard game theory, thus we do not go into the details of the workings of classical game theory here, we will simply provide a very brief historical overview to add context to our discussion.
The classical form of game theory that formed in the mid 20th century is the study of how to mathematically determine the best strategy for a given condition in order to optimize the outcome.[4] Game theory did not really exist as a separate field until John von Neumann published his article "On the Theory of Games of Strategy" in 1928. Von Neumann's original models became standard methods in game theory and mathematical economics. His paper was followed by his 1944 co-authored book Theory of Games and Economic Behavior, a foundational work for the domain of game theory that contains approaches for finding mutually consistent solutions for two-person zero-sum games.[5]

In the 1950s, the first mathematical discourse on the prisoner's dilemma developed, and an experiment was conducted. Parallel to this, John Nash developed a criterion for mutual consistency of players' strategies, now known as Nash equilibrium, applicable to a wider variety of games than the models by von Neumann and Morgenstern. Although game theory has come a long way since - being expanded in the coming years to deal with cooperative games, n-player games, evolutionary games and more - this work, and the assumptions that support it, today still form the bedrock of game theory. Anyone being introduced to the subject will be presented with two player game matrix and prisoner dilemma games followed by exercises in deriving Nash equilibrium for such games.

Maynard Smith famously wrote in the preface to Evolution and the Theory of Games, "paradoxically, it has turned out that game theory is more readily applied to biology than to the field of economic behaviour for which it was originally designed."[6] Smith's comments reflect the limitations of the classical modeling framework in dealing with real-world socio-economic phenomena. Game theory has been a valuable intellectual exercise that has offered much insight, but direct application of the theory of games to the solution of real problems has been rare.[7] Its chief uses have been to offer some insight and understanding into the problems of competition, and to provide mathematicians with new fields of interest; likewise, it has offered social scientists with some form of mathematical machinery with which to reason about human interaction. However, many important real problems exceed the bounds of the most developed versions of game theory and defy its basic assumptions.[8]

Since its origins game theory has been more of a theoretical exercise than an empirical tool. We can also note that it has been an analytical tool in nature.[9] In order to model human interaction with analytical methods only, game theory has had to make heroic and largely unrealistic assumptions about actors and the nature of social interaction as based fundamentally on dynamics of competition, between atomized individuals that are highly rational and optimize according to a single self-interested metric.[10]

Much of game theory has been focused around finding the equilibria of games that are then believed to predict how actual human populations will behave when confronted with situations analogous to the game being studied.[11] However, assumptions and predictions made by game theorists are often violated when applied to real world situations. Game theorists usually assume players act rationally, but in practice, human behavior often deviates from this model.[12] Empirical work has shown that in some classic games, such as the centipede game or the dictator game, people on a regular basis do not play Nash equilibria.[13]

These results reflect behavior issue in the analytical game theoretical framework. By using solely analytical methods the result is that the more theoretical and logically consistent we get the more limited in scope the theory becomes to real world application. The result of this is that we then have to often adopt more pragmatic approaches to deal with real world phenomena.[14] As Albert Einstein once famously said, "As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality."

This is illustrated within economics with the current divide between mainstream economics and behavioral economics. The increasingly theoretical and analytical approach to mainstream economics has led to a significant divide between the theory and empirical observation. This disconnect has proven fertile ground for the emergence of behavioral economics, with the result being the emergence of a "two system" framework. Likewise, the same dynamic exists between classical game theory and behavioral game theory.[15]

The essential problem is that analytical methods are closed form - meaning they model closed systems - when the situations that we are interested in, in the real world are very rarely closed. In particular, the more complex they become the more open they become and the less amenable to closed form analytical methods they become. Sufficed to say, standard game theory will work well in simple environments, where we have a finite amount of agents interacting in a well-defined fashion with limited interconnections and limited context in space or time, but it will struggle in more complex environments.

Expansion

Game theory should be a tool that helps us understand the socioeconomic world around us. If we want game theory to be that tool then we actually have to go beyond the analytical approach. For example, classical game theory may help us to analyze what is happening at one set of traffic lights, where agents are involved in a one off zero-sum game but it doesn't tell us much about the overall transport system within which this game is taking place and how to optimize across the entire system, this is a more complex problem involving a network of interactions between a whole population of agents.

In an age when connectivity and interdependence are growing; when collective action problems like climate change are growing; when the need for effective forms of governance are more important than ever, how can we make game theory a tool that really works to enable us to better understand real world dynamics of cooperation and competition. What we need today is a game theory of the real world, one that is able to maintain rigor but also deal successfully with empirical data from real world situations.

The answer to this lies in expanding the framework into the world of complexity. Classical game theory is based upon a set of simplified assumptions that enable it to gain traction and mathematical rigor on an inherently complex subject matter. But these simplifying assumptions are precisely what separates it from the real world, and limits its capacity to offer us valuable insight and understanding of real-world socio-economic dynamics of interest.

Developing such a framework that can deal with the complexity of the real world, will require revisiting the basic assumptions and expanding them to enable us to look outside the box of closed form games. It requires giving up the idea that the interactions between people in the world can be understood as closed systems with a single conception of value.

Game theory to date has largely dealt with static solutions of competitions and to a lesser extent situations of cooperation within closed systems. What it has not dealt with though is the context within which games take place. In this revision of the basic paradigm supporting game theory, games need to be thought of as primarily real world social interactions instead of mathematical machines for optimization. The focus needs to be on providing tools for studying real-world situations of interdependence between adaptive agents, instead of simply trying to convert these into mathematical problems of optimization.

Traditionally game theory has modeled games as essentially closed systems. That is to say that we isolate as specific sphere within which agents are interacting and are interdependent as they make their choices. But here it is important to start with the empirical phenomena that we are dealing with, and noting that many games of interest are not closed systems; games can be closed but they can also be open. Of course, no system in our universe can be said to be completely open or completely closed, it is certainly a matter of degrees. Whether a game can be considered open or closed - and thus best model using open or closed systems models - can be seen to be a function of how interconnected and interdependent it is with other systems both in time and space.

If we look at the traditional conception of the Prisoner's dilemma game then we can note that it is essentially a closed system. That is to say that the agents are isolated in space and time. They are separated from each other, they can not communicate and they do not share any social or cultural institutions. Likewise, the game is closed with respect to time, they have no experience of interacting with each other previously and the structure of the game is a given, it is not open to change over time. In such a case analytical models will work well, and that has been the focus of traditional game theory, to defined closed systems of interaction between agents - or at least find situations that were relatively closed and model them as closed systems. However many real world games can be said to be more open than closed. Most games do not take the form of two financial algorithms interacting in a market completely divorced from any other context. Instead many of the games we are interested in are multidimensional - such as the interaction of different cultures within a society, international politics or trade negotiations.

This has long been a critique leveled a game theory by sociologist and others in the social sciences - that it fails to capture the embeddedness of social reality. From this perspective when asked why game theory fails to capture empirical phenomena, many would simply posit that it is the framing of the situation as a closed system divorced from its social, ecological, cultural context that creates a false conception of the real world interaction of agents, which is always embedded within a context that makes it an open system.

It is only when one creates games that are closed in this nature that one actually gets the results that classical game theory might recognize. The tragedy of the commons may occur as classical game theory predicts once we have created a socio-economic situation that strips the dynamic of all context and creates a scenario for a closed model to be applicable. However in contrary to the predicted tragedy of the commons many societies for centuries managed to sustain their commons, and this was not because of highly sophisticated game theoretical models, but instead because of the fact that these games took place within a social-cultural-environmental context that enables them to be sustained in the face of collapse under pure competition and self-interest that a standard formulation of game dynamics as a closed system might predict.[16]

The generalization of game theory then needs to start with a more sophisticated foundational set of assumptions; not assuming that all games are inherently closed and then build models that are only relevant in such a context. Instead, a generalized version of game theory needs to starts with the recognition that games exist on a spectrum from being more closed to being more open and different frameworks will be required to deal with the different types.

Open games require a new set of tools from those used for modeling closed games. All most all of the assumptions that supported classical game theory need to be expanded upon when dealing with open games.[17] Firstly agents; the concept of the rational actor that has played such an important role in the formulation of games as closed systems needs to be expanded to recognize that decisions take place within a context that places limits on the degree of rationality that an agent has. Equally, there needs to be some recognition of the fact that people operate according to different value systems - as we will elaborate on below - not simply trying to optimize according to one single metric.

Secondly, an expanded version of game theory needs to recognize, as foundational, the many forms of cooperative interdependence that exist within most socio-economic dynamics. The formulation of the autonomous actor was a key part of enabling the desired end result of closed form models. The idea that the agent worked to maximize their payoff with limited regard for the cooperative structures that might limit this and reshape their decision making. Traditionally game theory has tended towards games of competition between autonomous agents based around zero-sum games; though not exclusively.

Thirdly to expand game theory into the real world of open games it is required to recognize that not all relevant information is contained within the confines of the game being played. Real-world games are often interconnected with and take place as part of large networks with the payoffs and dynamics of the game being interconnected with larger systems of coordination. Here we can already see network theory becoming a key tool that will enable us to look outside the game, as to what is happening in the environment and how that is affecting the game that is being played through positive or negative synergies.

Finally, a key aspect we identify in generalizing game theory will be in recognizing that any given game is often just an instance within a broader process that takes place over a period of time and which will inevitably condition and alter the features of any given game within that process.

This is a recognition that the rules to games are also open, that is to say, that in many scenarios the rules are not written in stone; people can create the rules for games and can change them.

Thus games are the product of an evolutionary process that has created the rule set and by increasing the iteration in the game the influences set itself can evolve rapidly to become an added dynamic element that needs to be accounted for not simply taken as a given. A recognition that in more complex scenarios there is a constant interplay between the agents in the game and the structure governing the rules of the game.

In the remainder of this paper, we will elaborate on each of these aspect that can be identified as presenting important dimensions along which our existing game theory framework will need to expand in order to meet the challenge of modeling more complex games and thus achieve a greater degree of functionality with respect to its real world application.

Behavioral Game Theory

In the preface to the influential book "Behavioral Game Theory" the author succinctly summarizes much of the problem with classical game theory when it notes that "the formalized study of strategy, began in the 1940s by asking how emotionless geniuses should play games, but ignored until recently how average people with emotions and limited foresight actually play games."[18] Game theory deals with the interaction between adaptive agents. Thus any game theory paradigm is going to have to give some account of how agents make decisions and act. Classical game theory is based on the conception of the rational agent, but over the past few decades, the formulation of the rational agent has come under mounting critique from many dimensions.[19]

The formation of the rational agent is manifestly not derived from a complete empirical observation, it is clearly a construction of the human being that is designed to deliver results that are amenable to standard mathematical modeling. As this model has been exposed to experimental reality many cracks have been made apparent.[20] The social science paradigm through which we understand human behavior is currently undergoing tectonic shifts as the 20th-century formation of human nature in the form of homo economicus is breaking down in the face of empirical studies and being replaced by a new conception of the social human being.[21] At the epicenter of this transition has been the area of behavioral game theory. Once heretical, behavioral economics is now fast becoming mainstream. Money managers employ its insights on the limits of rationality in understanding investor behavior and in taking advantage of stock-pricing anomalies. Public administrators use behavioral principles to boost participation in retirement-savings plans. Marketers now better understand why some advertisements and promotions attract consumers while others have little effect.[22]

A central element in the construct of game theory is the conception of rationality; that agents are taking actions to, in some sense, optimize according to some metric of success. It disequilibria formal models without the idea of rationality in some form. It is essentially this concept of rationality that gives some order to the system that we are studying, in that it tells us that some outcomes are better than others and agents will in some way search for those outcomes that are considered better.
Rationality in this context means that people choose their preferred option, the rational choice then is simply ascribing a value to one's payoff and attempting to maximize that payoff. This maximization of outcomes can then be modeled mathematically.[23] Payoffs represent how much an agent likes the outcomes to a situation. It is often this idea of rationality that is criticized, however, the limitations of game theory as we know it are not in this abstract concept of rationality they are more in the limitations of a narrow definition of it that has been constructed.

A game is defined by a mapping from various combinations of “strategies” taken by the involved “players” into resulting consequences in terms of “payoffs”. A “solution” predicts which outcomes of the game are to be expected. A major issue with traditional/neoclassical game theory, however, has been that its solution concepts, such as the Nash equilibrium or the strong equilibrium, rely on four rather extreme behavioral and informational assumptions on top of the idea of rationality.
Traditional game theory uses theoretical models to determine the most beneficial choice of all players in a game. Game theory uses rational choice theory along with assumptions of players' common knowledge in order to predict utility-maximizing decisions. It also allows for players to predict their opponents' strategies. Rationality is a primary assumption of game theory, so there are not explanations for different forms of rational decisions or irrational decisions.[24]
A recent paper[25] on the subject of behavioral game theory identifies four main assumptions in this theoretical construct. These are: 1. The joint strategy space is common knowledge. 2. The payoff structure is common knowledge. 3. Players have correct beliefs about other players’ behaviors and beliefs. 4. Players optimize their behavior so as to maximize their own material payoffs.

In the real-world, many of these assumptions are untenable. Players often do not behave like infallible optimizers in the sense of pure material self-interest, and it would be negligent to think of the resulting deviations as inexplicable irrationalities. Behavioral game theory then, seeks to modify these assumptions in various ways. Shaikh (2012) uses the term of “hyperrationality” to distinguish this concept of rationality in economics from a more general notion that actions and opinions should be based on reason - a broader view of rationality, as having sound reasons to do what it is that one does.

However, the agent of game theory, is one who is assumed to be not merely “rational” but also being “ultra-calculating”. The ultra-calculating agent is assumed to have unrestricted computational capabilities in their forward looking, their record keeping and calculation of payoffs. Further, an ultra-calculating rational agent will also assume that every other player is engaged in the same calculations, benefiting from the same computational capabilities. Added to this these agents have fixed and well-defined preference and a homogenous conception of value.[26]

The limitations of the rational agent are not hard to identify, the more challenging part is to distill the deviations into an alternative theory that is as accurate as standard theory and can be applied widely. For game theory to be a practical tool that helps us in dealing with real world situations of interest its basic assumptions about human nature will need to be grounded in empirical observation. In an expanded formulation of game theory it is not rationality per se that needs to be rejected - quite the contrary it forms solid foundations to game theory - it is the formulation of rationality that needs to be revised and expanded to make the model capable of accommodating empirical data.

As the physicist Murray Gell-Mann said, ‘Think how hard physics would be if particles could think.’ It is even harder if we do not watch what ‘particles’ do when interacting. Or, as Thomas Schelling (1960, p. 164) wrote, ‘One cannot, without empirical evidence, deduce what understandings can be perceived in a nonzero-sum game of maneuver any more than one can prove, by purely formal deduction, that a particular joke is bound to be funny.’ An expanded conception of the actors in game theory needs to start with data and remain strongly disciplined by data. This is exactly what the behavioral approach does, in that it is grounded in the empirical study of human behavior - too much so sometimes.

Behavioral game theory attempts to explain decision making using experimental data. The theory allows for rational and irrational decisions because both are examined using real-life experiments[27] Specifically, behavioral game theory attempts to explain factors that influence real world decisions. These factors are not explored in the area of traditional game theory but can be postulated and observed using empirical data. Findings from behavioral game theory will tend to have higher external validity and can be better applied to real world decision-making behavior.[27]

Game theory in standard experimental economics operates under the assumption of the rational homo economicus, while behavioral game theory extends standard (analytical) game theory by taking into account how players may value different things allowing for such ideas as cooperation or fairness, how their capacities for strategic thinking may be limited and altered through learning as well as the effects of learning.[28] Here we can identify two primary dimensions along which the conception of the agents within games needs to be expanded to create a more complex and complete representation of actors.[29] Firstly we will be looking at the idea of bounded rationality and then of utility.

An expanded concept of rationality needs to recognize that, people at the end of the day are not all knowing computers. How we process information and come to form beliefs values and expectations about the situations we find ourselves within is more of an imperfect evolutionary process rather than a perfectly rational one.

Hence, decisions are not best described by strictly maximizing behavior. In particular, the idea surrounding maximization are limited when players have incomplete information about the structure of the game and about the payoff consequences of different actions taken by themselves and others for the other players. Additionally, in a repeated game setting, players may be unable, or only partially able, to observe other players' actions and payoffs as the game unfolds. Hence, to describe more realistic human behaviors in complex game settings, models of boundedly rational behavior, possibly allowing for learning dynamics, are necessary.[30]

Something that we can emphasis here is observation, that while human actors have bounded factual knowledge and computational capability, they do have an extraordinary wealth of social knowledge about social and cultural dynamics: in particular, their knowledge of diverse cultural forms and institutions such as family, government, business, market or work organization, which they apply in their social relationships and game interactions.[31]

One of the main research directions in the behavioral research tradition comes under the heading "bounded rationality" dealing with the consequences of limited human rationality. The easiest way to explain this concept is with a game. Bounded rationality can be best illustrated with reference to a simple experimental game. The game consists of a group of 100 people with each person picking a number between 0 and 100. The goal is to pick the closest number to 70% of the average of all the numbers chosen. The predicted Nash equilibrium for this game is for everyone to pick 0. In studies, the median guess is around 35 and the winning guess is generally around 25. The game illustrates the phenomenon of bounded rationality, in that it displays the limits of how far normal people will carry out a process of reasoning before terminating it.

Generalized game theory[32] is one approach that tries to build models based on this assumption of bounded rationality and the need for a more complex representation of agent psychology in the modeling of games. Here games are constructed of what are called rule complexes, which is a set containing rules. However, these rules may be imprecise, inconsistent, and even dynamic. This is a generalization of a set of rules and provides a tool to investigate and describe how rules can function as values, norms, judgmental or prescriptive rules, and meta-rules.[32]

Also possible is to examine objects consisting of rules such as roles, routines, models of reality, algorithms, social relationships, and institutions. Rule complexes are especially associated with sociologist Tom R. Burns and Anna Gomolinska and the Uppsala Theory Circle. Proponents of generalized game theory have advocated the application of the theory to reconceptualizing individual and collective decision-making, resolutions of the prisoners' dilemma game, fuzzy games, agent-based modeling, conflict resolution procedures, challenging and providing robust and normatively grounded alternatives to Nash equilibrium and Pareto optimality, among other insight that it may offer.[32]

Because its premises derive from social theory generalized game theory emphasizes and provides cultural and institutional tools for game conceptualization and analysis, what Granovetter (1985) refers to as the social embeddedness of interaction and social and economic processes. The modeling of the actors themselves in generalized game theory is especially open to the use of concepts such as bounded rationality and incomplete information.[32]

The second strand of explanations for behaviors that consistently contradict equilibrium predictions based on the standard assumptions of self-interest and unbounded rationality may be that players are guided by alternative preferences. In other words, provided the assumption of material self-interest is flawed and that, instead, higher-order motives such as altruism or social norms guide a player's actions, then self-interest predictions are misguided, even if players follow maximizing behavior. It is not that players' behavior is not maximizing, rather what they are trying to improve may not be some object that is associated with narrow self-interest.

Thus the default assumption of fixed preference and utility needs to be reconstructed in an expanded formulation of game theory. Rationality is typically understood to mean that people only have one conception of value - such as money - and are simply trying to optimize according to that metric without regard for other factors, that their interests and desires are exogenously given and do not change over time.[33] This conception of value needs to be expanded to reflect the different types of value that people try to pursue in their lives and how they can change over time, based on the individual and changes in context.

When players are involved in a voluntary contributions game, robust evidence shows that many agents often contribute substantially even when free-riding is the strictly dominant strategy. The Ultimatum Game is a classic example of research that illustrated this and uncovered violations of these standard assumptions of utility and rationality. In the experiment, one player (the proposer/allocator) is endowed with a sum of money and asked to split it between him/herself and an anonymous player (the responder/recipient). The recipient may either accept the allocator's proposal or reject it, in which case neither of the players will receive anything. From a traditional game-theoretic perspective, the allocator should only offer a token amount and the recipient should accept it. However, results showed that most players offered more than just a token payment, and many went as far as offering an equal split. Some offers were declined by recipients, suggesting that they were willing to make a sacrifice when they felt that the wherein did not accord with some conception of unfairness.[34]

Such observed empirical data clearly runs contrary to a reductive interpretation of game utility maximization but can instead be model within an expansive conception of value where agents are trying to operate according to a number of different forms of value - in this example above, social or cultural capital. Ultimately, an in-depth examination of what specific elements of reasoning, psychology, and environment guide an individual's actions is likely to be needed to generate a robust explanation. Shortcut assumptions about optimization according to a single metric may work in simple situations but will likely not be sufficed for dealing with a more complex world.
Generalized game theory conceptualizes game equilibria as an outcome of an optimization over a heterogeneous value set, instead of a single homogeneous conception of utility. The outcome of such an expanded model would give rise to a different point of equilibrium that would better likely match what we see in such a social dynamic as the ultimatum game.

Cooperation & Competition

“I feel, personally, that the study of experimental games is the proper route of travel for finding 'the ultimate truth' in relation to games as played by human players. But in practical game theory the players can be corporations or states; so the problem of usefully analyzing a game does not, in a practical sense, reduce to a problem only of the analysis of human behavior. It is apparent that actual human behavior is guided by complex human instincts promoting cooperation among individuals and that moreover there are also the various cultural influences acting to modify individual human behavior and at least often to influence the behavior of humans toward enhanced cooperativeness." - John Nash

A central question of interest across the social sciences, economics, and management is the question of how people interact with each other. Humans live out their lives as social creatures and the outcome to our individual lives and the social systems that we form part of - whether they are families, businesses, governments, economies or whole society - is largely a function of the nature of our interaction with others.
Of course, human beings and social interaction is a very complex phenomenon, we see people form friendships, trading partners, romantic partnerships, businesses compete in markets, countries go to war, the list of types of interaction between actors is almost endless. In developing models for such situations probably the most fundamental question we can ask is do the actors involved interact in a constructive or destructive fashion? In an informal fashion, a constructive interaction is what we call a cooperative relation and a destructive relation is one that we would term competition or conflict. In slightly more formal terms this can be called a positive or negative synergy.[35]

Real-world interaction between people in multi-agent systems involves elements of both cooperation and competition. This parameter of positive and negative synergy or cooperation and competition is fundamental to social interaction in that it tells us the overall structure to the game being played. Social interaction takes many different forms, but the difference between a cooperative and competitive interaction is of primacy in that it is a qualitative difference in the nature of the game that is being played.

Game theory has its origins in situations of competition, the classical application being the cold war interaction between the US and the USSR. Non-cooperative games are greatly more amenable to traditional tools of mathematics, in that they often involve zero-sum games and thus deliver linear results and equilibrium outcomes. In most courses on game theory, non-cooperative game theory is presented as the default position, for the understanding of interaction between agents. Much of game theory loses its predictive capacity without the idea of competition between agents.[36] The fact that the theory of von Neumann and Morgenstern says little about coordination problems, in particular, and variable-sum games, in general, illustrates the issue that the guiding hand of self-interest becomes weak in strategic situations if there is no conflict of interest and players have difficulties in forming expectations about the behavior of their fellow players.[36]

Baumol summarizes the classical view on game theory which derives from the minimax theorem: "In game theory, at least in the zero-sum, two-person case, there is a major element of predictability in the behavior of the second player. He is out to do everything he can to oppose the first player. If he knows any way to reduce the first player's payoff, he can be counted upon to employ it"(1972, p. 575). However, the minimax theorem loses its power if the players' interests do not contain pure conflict and the zero-sum modeling becomes inappropriate.[36]

Game theory as we have known has largely been a mathematical tool for analyzing and predicting how humans behave in strategic situations based largely on the idea of equilibrium. Standard equilibrium analyses assume that all players: 1. form beliefs based on an analysis of what others might do (strategic thinking); 2. choose the best response given those beliefs (optimization); and 3. adjust best responses and beliefs until they are mutually consistent (equilibrium).[36] An alternative way to define the equilibrium condition is that players are never surprised when the strategies of other players are revealed.
The default position of non-cooperation is a critically important element in that it helps to formulate the central idea of classical game theory; that of equilibrium. The basic intuition of the Nash equilibrium is in predicting what others will do and then choose your optimal strategy given that. When everyone is choosing their optimal response, we get a Nash equilibrium and it is believed that we can predict games by finding these points of equilibrium.

These ideas extend even to include questions of how cooperation may emerge even in the presence of self-interested behavior. Classical game theory searches for competing interests that interact and counterbalance each other to create an equilibrium, whether that is in dynamics of cooperation or competition. Even systems that one would conceive of as cooperative or collaborative are modeled in this fashion. For example, it has also been proposed that game theory can explain the stability of any form of political government. For example, we can take the case of a monarchy, the king, being only one person, cannot maintain his authority by personally exercising physical control over all or even any significant number of his subjects. Sovereign control is instead explained by the recognition by each citizen that all other citizens expect each other to view the king as the person whose orders will be followed. Coordinating communication among citizens to overthrow the sovereign is effectively barred, as conspiracy to replace the sovereign is typically punishable as a crime. Thus, in a process that can be modeled by variants of the prisoner's dilemma, during periods of stability no citizen will find it rational to move to replace the sovereign, even if all the citizens know they would be better off if they were all to act collectively.[37]

Classical game theory assumes a social structure where the actors are “autonomous” or independent from one another. Each actor judges the situation in terms of her own desires or values. There is no concern with others as such. This is illustrated by the classical rational agent who assigns values or preferences to outcomes and the patterns of interactions in terms of their implications for herself – and only herself – and tries to maximize her own gain or utility.[38]
The empirical fact that subjects in most societies contribute anything in the simple public goods game, is a challenge for game theory to explain via motives of total self-interest. However one of the defining features to human beings is their extraordinarily high level of cooperative behavior.[39] Cooperation is a massive resource for advancing individual and group capabilities, and over the course of thousands of years, we have evolved complex networks for collaboration and cooperation which we can call institutions of various form. These institutional structures help us to solve the many different forms of the tragedy of the commons that we encounter within large societies.[39]

Noncooperative games are an exercise in studying the optimal strategies that players should follow and where they intersect to create equilibrium points. But this fails to answer any questions about the nature of the whole organization, how we get optimal outcomes on the macro-level. Equilibrium analysis of social interaction has been an effective and powerful tool that gives us much traction and delivers great value in understanding the basic structure to social systems. However, it should not be the only tool we use. In order to understand cooperation it is important to start with the recognition of interdependence, as an inherent and critical part of social reality, and how this leads to makes, the result being a very different model.

This narrow conception of social relationships presented by equilibrium analysis will certainly not be sufficed to deal with the complexity of real-world dynamics of interdependence. Actors are not only interdependent in action terms but in social relational, institutional, and cultural-moral terms. Thus, the importance of taking into account and analyzing such factors as the social context of games – which contribute to defining many of the rules of the game.[40]

As Lewin (1948) noted: "The essence of a group is not the similarity or dissimilarity of its members, but their interdependence. . . . A change in the state of any subpart changes the state of any other subpart. . . . Every move of one member will, relatively speaking, deeply affect the other members, and the state of the group." (pp. 84–88) Games in the real-world take place within social and cultural cooperative structures that have evolved over millennia. Certain games create attractors towards competition while other games create attractors towards cooperation. How cooperative or noncooperative a game is, can be seen to be a function of both the structure of the game and the degree of interconnectivity between agents involved. Some games are inherently competitive - games of rivalrous excludable goods for example - while others are not. These factors are long since known and studied in the social dilemma, tragedy of the commons, public goods games and such models. Other games engendered attractors towards cooperation, both in their structure - being non-zero sum and nonrivalrous - and in the capacity for agents to form cooperative structures through communications.

The study of game theory has been largely grounded in the assumption of autonomous actors, when in the real-world people typically find themselves within a context of interdependence. Interdependence is inherently non-linear, with such nonlinear phenomena one can not understand them by simply looking at the individual parts but one has to look at how the parts interact and the whole system. In looking at games of interdependence we need to look at the overall game, and how this overall outcome emerges out of the interaction between the parts. More specifically this means looking at the effect that each has on the other, that is to say, the nature of how the players are interdependent. The nature of these dynamics of interdependence is well understood within the area of social interdependence theory.[41]

Within social interdependence theory, the essence of a social system is seen to be the interdependence among members, which results in the group being a dynamic whole so that a change in the state of any member or subgroup changes the state of others. Group members are made interdependent through common goals. In social interdependence theory, the nature of the interdependence between two individuals is contingent upon the manner in which each can influence what happens to the other during the course of the social interaction; where this is called 'outcome interdependence'. The basic premise of social interdependence theory is that the way in which goals are structured determines how individuals interact, i.e. the types of relations between them. The theory posits two different types of social interdependence, positive and negative.[42]

Interdependence creates relations of competition and conflict, conflict arises when agents' agendas are mutually exclusive, in order to get this dynamic, agents must be acting under different agendas over the same rival goal, the agents must be acting under their own personal agenda or that of a group that the other agent or agents are not acting on behalf of. Conflictual relations define boundaries around the personal interest of an agent and the conflict is always between the agents that are internal to the boundary and those external to it, as such, they work to define borders and differences between social actors.
Positive interdependencies are typically built up around some shared function that requires more than one person to perform, an example might be two people carrying a table that is too heavy for either in isolation, in order to achieve the combined outcome each role has to be fulfilled, thus for any agent to obtain the joint outcome they must be as equally interested in their own function as that of others. This dynamic of cooperation creates a positive sum game. In isolation neither person could move the table, thus when we simply added both of our individual actions in isolation we would get nothing, when we combined our activities though we got something that was more than the sum of its individual parts, the table was moved, by cooperating we added value to the whole system thus creating a positive-sum game.

The word synergy means a construct or collection of different elements working together to produce results not obtainable by any of the functions alone. The value added by the system as a whole, beyond that contributed independently by the parts, is created primarily by the relationship among the parts, that is, how they are interconnected, thus things have to be interrelated in a particular fashion.[42] Synergistic relations are ubiquitous in our world, physical, biological and social, they involve both differentiation and integration where components are different and working together they complement each other and the combined effect is greater than the sum of its parts. A song is a good example of a cultural synergy, taking more than one musical part and putting them together to create a song that has a much more dramatic effect than each of the parts when played individually, the song as a whole exists out of the interaction between the different instruments, but we only get this emergent phenomenon of the whole song by each individual musician coordinating their activity with that of others, if they are not coordinated we will just get a bad noise.

The atomized view of social reality that classic game theory is grounded upon leads to the social dilemma and the tragedy of the commons. A long since noted element in the study of noncooperative games, like the prisoner's dilemma, is the paradox that the cooperative outcome is better for everyone than only the Nash equilibrium. The equilibrium is not "Pareto-optimal" (efficient in an economics sense). A solution is Pareto-optimal if the only way to achieve a better payoff for one player is to give a worse payoff to another player. Achieving cooperation in the prisoner's dilemma proves to be a difficult and relevant problem one that can not be resolved by looking inside the box of non-cooperative assumptions but requires us to recalibrate and reformulate the model in terms of interdependence and the cooperative structures that emerge out of them.

Cooperative structures are the various forms of institutional structures that enable people to act cooperatively, examples being third party organizations like governments, social networks of, form and recommendation systems. How we go about solving the social dilemma depends on the degree of interconnectivity and interdependence within the dynamic. At a low-level, cooperative structures have to be imposed through regulation, while at a high level this is no longer necessary as the interconnectivity and interdependence can be used to create self-sustaining cooperative organizations. This is illustrated by how different cooperative structures have evolved within society, those within small closely interdependent groups like the family that are informal and those that have evolved for larger society that is composed of many groups that are more independent.

Traditionally game theory - that emphasizes autonomous and self-interested individuals while downplaying the existence of the interdependence between actors - has often led us to the conclusion that in order for optimal overall outcomes to be achieved it is necessary to impose them. The cooperation requires some kind of exogenous factor - typically in the form of governments. However, by reformulating the assumptions away from the atomized individual and focusing equally on the embedded set of interdependencies that people often find themselves within we can identify that interdependence as a potential source for enabling cooperation.

When interconnectivity between members within a game increase, so typically does interdependence and this changes the nature of the game. The externalities that create the social dilemma are things that we can put external to our domain of value and interest, but interconnectivity reduces the capacity to do this. An externality is not necessarily something that is far away, it is simply whatever you exclude from your value system so that reducing it has no reduction to your payoff. But connectivity takes this barrier down requiring us to recognize the value of the other entity and factor it into our decision. This connectivity can be of many different kinds. Communication is a form of connection that can enable positive interdependence and there is a robust finding in the social dilemma literature that cooperation increases when people are given a chance to talk to each other.[43] Cooperation generally declines when group size increases. In larger groups, people often feel less responsible for the common good; as they are more removed from it and the other people with whom they share it.

Thus we can see what is really at the core of the social dilemma is the question of what people value and how far that value system extends. Wherever we stop seeing something as part of us or our group, that is where negative externalities accumulate and start to give us the social dilemma. However, by building further connections so that people recognize their interdependence with what they previously saw as external, they will start to factor it into the value system under which they are making their choices and reduce their negative externalities. From this perspective, the issue is really one of value and externalities. Connectivity can change that equation, working to internalize the externalities. Connectivity though is just an enabling infrastructure, one still has to build the channels of communication and structures that enable positive interdependence.

Building systems of cooperation in such a context means enabling ongoing interaction, with identifiable others, with some knowledge of previous behavior, lists of reputations that are durable and searchable and accessible, feedback mechanisms, transparency etc. These are all means of fostering positive interdependence once interconnectivity is present and through them, self-regulating and sustainable systems of cooperation can be formed.[44] In the public goods game, if the amount contributed is not hidden, then players tend to contribute significantly more. This is simply creating a transparent system where there is feedback. As another example, we could think of eBay. EBay is really a huge social dilemma game; one would not send money before receiving the item nor would the other party send the item before receiving the money, so why has eBay succeeded? Not because eBay throws people into jail if they do not adhere to the rules, it is because of communication, transparency and feedback mechanisms that build positive interdependence.

An expanded formulation of game theory needs to place equal emphasis on the autonomous individual as on interdependence, and on cooperation as competition. This change in paradigm would enable us to move beyond the narrative of the tragedy of the commons and begin to recognize the key aspect that real world interdependencies between actors plays within games. This is best illustrated by the work of Elinor Ostrom.[45]

Network Game Theory

Game theory has been focused on individual games as closed systems wherein a limited number of agents interact, with that interaction dependent upon the rules of the specific game and the individual agents involved. However, as we move into a world of complexity; as socio-economic systems become more interconnected within large complex systems, what we begin to see is really the factors determining a game may not be fully bounded within that game. Instead, the specific game is taking place within larger socio-economic networks and the payoff and nature of the game become contingent on factors taking place in other parts of the network; or even the structure of the whole network. For example, in traditional game theory, we might try to analyze the trade relations between two nations as a closed system where all relevant factors are contained within the closed game. However, as the world becomes more interconnected more factors within that negotiation will come to be determined by other factors or games taking place across the network, what other nations are choosing to do, how the whole global economy is changing over time etc.
As people come to form part of ever larger systems of organization their payoff for performing some action becomes increasingly interconnected with other games being played and a closed form of model becomes less relevant. Thus we can identify network game theory as another dimension to expanding the game theory framework to deal with the empirics of complex socioeconomic systems.

In large cooperative systems, interoperability becomes increasingly important, for example, the payoff to an individual for choosing a particular computer operating system comes to depend on what computer operating system their colleagues are using; the choice of what social network to join depends on which one your friends are using etc. To illustrate the difference between the closed form of classical games and the open form of networked games we can take crime as an example. Crime has long since been analyzed in game theoretical terms of costs and benefits to the criminal for performing a particular crime. Crimes such as smuggling goods across borders can be thought of as games of cat and mouse between the criminals and the police.

But equally, crime can be analyzed as a network game by looking at the structure of the social network between criminals and how this affects the norms and expectations of the criminals, given them a greater or less propensity to commit a crime. Criminal activity is, to a certain extent, a collective or group phenomenon, and the crime and delinquency are related to positions in social networks.[46] Indeed, delinquents often have friends who have committed offenses, and social ties are a means of influence to commit crimes. Glaeser, Sacerdote, and Scheinkman (1996) were among the first to look specifically at criminal social interactions. Their model clearly and intuitively shows how criminal interconnections act as a social multiplier on aggregate crime.

This helps to illustrate how the study of complex socioeconomic systems will require effective tools for modeling whole networks of games and how they interact. In such circumstances simply trying to pick out one game, isolate it and model it as a closed system with a limited number of parameters will only give us a very partial understanding. The main challenge that is faced in studying strategic interaction in social settings is the inherent complexity of these networks.[47] Key questions here include; how do the actions and games being played on the network interact with each other? How does the structure of the network effect the individual games being played? Or how do events that are spreading across the entire network effect individual games?
Coordinations become a key aspect when looking outside the box of any given individual game to see how the actions of others may affect the payoffs and choices made within the game of interest. In economics and game theory, the decisions of two or more players are called strategic complements if they mutually reinforce one another, and they are called strategic substitutes if they mutually offset one another. These terms were originally coined by Bulow, Geanakoplos, and Klemperer (1985). It is an important class of games because of its many applications; including the choice of a language, a technology, whether to buy a new product, adopt a particular behavior, and so forth.

Such interactions are natural ones to model using game theory, as the payoffs that an individual receives from various choices depends on the behavior of his or her neighbors. In games of strategic complements, an increase in the actions of other players leads a given player's higher actions to have relatively higher payoffs compared to that player's lower actions.[48] Examples of such games include things like the adoption of a technology, human capital decisions, criminal efforts, smoking behaviors, etc. Games of strategic substitutes are such that the opposite is true: an increase in other players' actions leads to relatively lower payoffs to higher actions of a given player. Applications of strategic substitutes include, for example, local public good provision and information gathering.

Another classical example of a game on a network is based on what are known as "bestshot" public goods games.[49] For instance, the action might be learning how to do something, where that information is easily exchanged between people; or buying a piece of music that is easily lent from one player to another. Taking the action is costly and if any of a player's neighbors takes the action then the player is better off not taking the action; but, taking the action and paying the cost is better than having nobody in a player's group take the action.

Network game theory can help us analyze games, payoff and actors behavior in terms of the overall structure of the network where in the game is being played. Here primary considerations include such things as network density, relating players' behaviors to their position in the network, and relating behavior to things like the degree distribution and cost of taking given actions. The theory can help us to understand a player's behavior in relation to his/her position in a network, as well as what overall behavior patterns to expect as a function of the network structure.[50]

More specifically one could think of the example of how power influences socioeconomic games and take a network approach to analyzing this. Power is something that is often not accounted for within standard game theory but is, of course, a critical aspect of many real world games. The many disequilibrium in power that exist in real-world games - and that strongly affect them - are largely not accounted for in traditional game models. Power can in many cases be understood in terms of connectivity i.e. the more social, cultural or economic connections one has the greater the person's position within a game.

The main concept is that the act of exchange depends on the agents' other opportunities and their environment, and thus getting a deeper understanding is possible only by examining these factors. The position of a given agent in a network, for example, can endow her with power over the auctions and deals she makes with her partners. For example, if we were to analyze solely the military relationship between Japan and North Korea in isolation as a game, we could note that in may aspects North Korea has a much larger army; in terms of troops, navy, tanks, missiles. However if we take account of the military network of alliances this game would change significantly in that North Korea has few connections and allies, while Japan has many including the largest military force on Earth. The closed game and the open network game here look very different and it would be important to understand this network of connections in order to understand the real-world dynamics of the game.

Likewise, we could think about a trading relationship between two businesses, where if one had more partners and sources than the other they would be in a much stronger position for negotiation. The degree of connectivity to an actor within a game can tell us how many other options the player may have and thus the balance of power within that game. Equally, a company could have a privileged bargaining position in a game due to its role within a larger supply chain where it functions as the only bridging link to the flow of important goods or commodities. Network theory is a key tool for studying the architecture of complex systems. By combining game theory with network theory it can be a powerful tool for formally modeling the game dynamics within complex social systems. This would again help us to move away from the closed model and towards an approach that can begin to capture and formalize our reasoning about the embeddedness of many complex games, and how that affects the game being played.

Dynamical Games

Game theory started out life as the study of static games; the single interaction between non-cooperative actors as exemplified by the once off prisoner's dilemma game, such isolated events can be understood well as closed games without context in time. However much of real world interactivity exists within the context of a larger process of ongoing interaction that has created the current context for the game and in such circumstances it can become critical to understand that larger process. In this respect the development of evolutionary game theory during the 70s was an important breakthrough for game theory and a step in the direction of it becoming more dynamical and capable of dealing with multi-agent systems.

Many strategies that agents play and even the rules of the game that they are playing are not the product of some abstract rational process of reasoning that created them, but instead the outcome of an ongoing process of evolution - one the has both phases of equilibrium but just as importantly phases of non-equilibrium as the population interacts and explores its phase space of possibilities. One of the most interesting developments that biological evolutionary game theory introduced was that it no longer need the construction of the rational agent. Evolutionary models assume that people choose their strategies through a trial-and-error learning process in which they gradually discover that some strategies work better than others.[51] In games that are repeated many times, low-payoff strategies tend to be weeded out "naturally" and an equilibrium may emerge, without the need for the hyper-rational agent. Indeed because this process was taking place on a population of creatures like birds or even plants one did not need to even talk about rationality at all, it was essentially built into the structure of the game.[51]

In the context of evolutionary biology, the two basic notions of game theory, namely strategy and payoff, have to be re-interpreted. A strategy is not a deliberate course of action, but an inheritable trait. The payoffs are interpreted in terms of Darwinian fitness i.e. degree of average reproductive success. The 'players' are members of a population, all competing for a larger share of descendants.[52] In an economic context, the same theory describes population changes because people play the game many times within their lifetime, and consciously - and perhaps rationally - switch strategies to those that have proven to work best. Binmore propounds the idea that social evolution develops consistent rules of fairness which can be used by people to solve the coordination problems of a society.[53] If there are competing societies which are more successful in solving the coordination problem, then one society is likely to have to adapt in choosing different rules and thus becomes a different society or simply becomes extinct.

In this model then the rational or logic that is giving some order to the game is not seen to derive from the individual agents in the game but instead comes more from their interaction over time and their adaptive capacity. In this case we do not need to make any assumptions that the agents know automatically the optimal strategy of how to play a game, but instead the optimal strategy may be seen as the outcome of an ongoing learning process, where those that adapt become more prevalent, those that do not die out, in such a way the logic emerges out of the interaction between agents and between the system and its environment. This adaptive learning process would appear much more credible source on which to base the logic of game theory as opposed to the hyper-calculating rational agent who seems to know everything through reason alone.

On a socio-economic level, evolution is synonymous with learning rather than genetic selection: applying new behavioral concepts, new ways of thinking and forms of social interaction. This is a result of exposure to repeating phenomena and imitating other individuals who are in general more successful - instead of calculating best replies. In such a case, "players need know only what was successful, not why it was successful"[54]. Successful behavior thus becomes more dominant within the society not only because, for instance, market competition selects against unsuccessful behavior, but also because agents imitate successful behavior.[55] These ideas have since been captured in the model of the replicator equation.[56]

Evolutionary models can enable us to reason about the state of a game, or the out come to a game, that remains in a state of non-equilibrium (suboptimality) without having to, by necessity, converge to the optimal equilibrium. Evolutionary game theory combined with computer simulation tools could act as a "post-rational actor" approach to game theory that enables us to explore the full set of possibilities to a game, both equilibrium, and non-equilibrium, and to reason about the possibility of there being multiple equilibria for any given game.

Additionally, evolutionary game theory sees game equilibrium as the outcome of an adaptive learning process and selection, rather than as the result of strategic reasoning by rational agents. This idea has been given some structure through two sets of solution concepts namely: the evolutionary stable strategy and a set of dynamic concepts which examine the stability of the evolutionary process.[57] The basic idea is that people will move away from Non-equilibrium solutions and that we can expect non-equilibrium to disappear over time. In a sense, this is a form of short cut where to analyze a game we only need to know the equilibrium outcome. Evolution though is notorious for producing suboptimal outcomes. To fully understand games it is really necessary to understand the full state space where in the system can potentially exist. When we begin to recognize that not all evolutionary processes lead to a single equilibrium we can begin to recognize the importance of non-equilibrium states in games.

Subjects' thinking in initial responses to games tends to avoid the fixed-point or indefinitely iterated dominance reasoning that equilibrium often requires. In many games this makes their decisions deviate systematically from equilibrium, in such a case the system may converge upon equilibrium overtime but more as the outcome of an interactive learning process rather than one of static deliberation. Several recent experimental and empirical studies suggest that people's initial responses to games often departs systematically from equilibrium, and that structural non-equilibrium "level-k" models often out-predict equilibrium. Even when learning is possible and converges to equilibrium, such models allow better predictions of history-contingent outcomes.[58]

Equilibrium analysis largely just tells us about a single optimal outcome, when in the real world this is often not what we see. Thus it is important to also look at non-equilibrium states and the potential of a game to have multiple basins of attraction. An evolutionary computational approach can help us to explore not just the stable equilibrium outcomes but also the non-equilibrium processes that arrive at such points and how games may converge upon suboptimal outcomes due to history - what is called hysteresis, the dependence of the state of a system on its history. The result of such modeling would give us a richer understanding of games.

In their classic work, Von Neumann and Morgenstern defined a game as simply the sum total of the rules which describe it. They did not, though, develop a theory of rules. In standard game theory, the rules of the games are considered largely fixed, with the agents then making their choices within these rules. However a key aspect of complex systems is the coevolution of agents and structure; structures are not given but instead, agents create the structures which then feedback to condition the actions of the agents in an ongoing process of feedback between the micro and macro levels of the system.

With evolution, the construction of the game and thus the payoffs to the agents can change. Evolutionary game theory considers games involving a population of decision makers, where the frequency with which a particular decision is made can change over time in response to the decisions made by all individuals in the population. As such it is inherently dynamic, both on the micro level but also on the macro level.

For example in the classical game experiment of Tit for Tat[59] one can see how as the mix of the strategies within a population of agents changes so too does the success of a given strategy. The unconditionally cooperative strategy does not work within a population of unconditional defectors, it is only when a population of generous Tit for Tat agents has taken hold that the unconditional strategy will become successful. This illustrates how in dynamic games of whole populations there is a feedback dynamic between agents and structure and how the structure or rules of the game themselves evolve to create new context wherein new possibilities are viable.[60]

Another thing to note here is that, social evolution does not necessarily follow the dynamic patterns of genetic evolution studied in biology. Learning can be fast, especially when it is in the form of imitation, and communication can allow for almost immediate changes on a large scale and even more so in a world of increased interconnectivity. In such a context of rapid iteration, new structures can emerge making previously unviable scenarios viable and vise versa. In extending game theory to the world of complex social systems evolutionary dynamics need to be incorporated to capture this important feature. Generalized game theory has developed the theory of combining, revising, replacing, and transforming rules and rule complexes.[60]

Within generalized game theory, the agents may have the capacity to transform game components, either the individual role components of one or more players or the general “rules of the game”. Rule formation and re-formation is, therefore, a function of interaction processes. With open games, the players may restructure and transform the game and, thereby, the conditions of their actions and interactions.

Conclusion

The study of complex systems challenges our existing scientific framework along many dimensions, but the limitations of traditional models based on linear systems theory when faced with complex systems, is probably made nowhere more explicit than within economics and game theory. In the past when linear models have bumped up against the complexity of the real world we have tended to brush the data - that did not fit - under the carpet and stick to the theory, but this is becoming increasingly untenable. With the rise of behavioral economics, the genie is out of the bottle and critiques of the rational agent are no longer marginal but becoming increasingly mainstream. Added to this is the rise of complexity science and the increasing demand for models that can deal with the complex systems that we are increasingly coming to recognize as inherent to our social, natural and engineered world.

Developing new models that can better reflect how people really interact is of great importance not just in the social sciences but also in society at large. Conventional game theory is an important foundation to game theory but clearly not sufficed for such an endeavor. Equilibrium analysis is an important tool but the framework clearly needs to be extended along a number of dimensions in order for it to develop and grow in the coming decades. This paper has been an outline of some of the major trends and themes that we see developing and that will contribute significantly to the development of that much needed tool of a general theory of complex games that remains somewhat elusive.

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